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A CRITICAL STUDY OF THE DEVELOPMENT OF ELEMENTARY MATHEMATICAL PEDAGOGY IN MASSACHUSETTS

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A CRITICAL STUDY OF THE DEVELOPMENT OF ELEMENTARY MATHEMATICAL PEDAGOGY IN MASSACHUSETTS

Bernardine Joan Cooney

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF MASTER OF SCIENCE AT

MASSACHUSETTS STATE COLLEGE

1936

The writer wishes to express her sincere appreciation for the help and encouragement extended to her by the following members of the faculty:

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INTRODUCTION

On first consideration, A Critical Study of the Development of Elementary Mathematical Pedagogy in Massachusetts may seem to be interesting but unimportant because of its emphasis on the past. Many of us with our minds entirely in the present do overlook the importance of the last few centuries in the moulding of our institutions. This is a natural attitude. However, in our schools we are, whether we like it or not, transmitting the heritage of the past. If we are to judge what part of that heritage is important, we must know what has been considered noteworthy by educators of the past, and the reasons for their preferences. Then we can decide what is fitting for our own society.

The writer first tried to develop a mathematical pedagogy by reading modern works in the field. Because of many conflicts and contradictions therein, she found this process very confusing. The most intelligent solution of the problem seemed to be an historical analysis of the theories and practices in the teaching of mathematics during the last three hundred years. This period has been divided somewhat arbitrarily into seven sections: (1) The Colonial Period (1630-1776), (2) The Transition (1776-1820), (3) Pestalozzi (1820-1837), (4) Horace Mann (1837-1880), (5) The Beginnings of the Scientific Attitude (1880-1910), (6) The Scientific Era (1910-1925), (7) Summary and Conclusions. Throughout there has been an effort to analyze social and educational conditions which seemed pertinent to the development of mathematics in the schools. This has been followed with a discussion of the subject matter used and the method of its presentation.

Many old texts and periodicals have been consulted, as well as works by representative men in the field of mathematical pedagogy. The work is necessarily incomplete, but has been most beneficial to the wr ter. Let us hope that it may be to some few others.

THE COLONIAL PERIOD

i.

Some Important Influences

The Massachusetts Bay Colony was founded in 1630. Every child upon entering our public schools has the enormous importance of that fact impressed upon him. This emphasis takes place because it has long been considered the proper task of our educational system to idealize the founding of our country. Consequently, inconsistencies and half truths have crept into the average American's conception of the early history of this nation. Considering this fact, the writer thought it excusable to review some important influences on life in Massachusetts from 1630 to 1775.

Probably the three greatest factors in the pre-revolutionary history of the Massachusetts Bay Colony were:

1. The geographical environment.
2. The Puritan movement in England.
3. The Mercantile Theory.

Undoubtedly, it was the first two which had such different bearing on future educational development, so the writer will discuss these somewhat in detail.

The story of the early beginnings of any people is usually that of a series of struggles against the environment and the tale of its effect upon them. This was no less true in New England. Here there is a very severe climate where, even at the present time, it is hard to withstand the winters. It was much worse around 1630, when men were trying to convert a wilderness into a home. There was little time for frivolity or any patronage of the arts. One is wont to speculate as to what would have become of our first settlers if they had colonized Virginia or Georgia instead of the land in the North. Would such a pleasing climate have lessened their environmental struggles and led to an early flowering of American culture? Such a result might seem to be logical, but many feel that the tendency toward easy living would have presented a conflict to the fundamental spirit of the Puritan.

In nearly all the books published prior to 1910 concerning early New England, our Puritan fathers are described as models of propriety and learning. Let us proceed to look into the matter in detail. The term Puritanism is applied to both Separatists and Non-conformists because, although they had the different historical backgrounds, there developed in both these sects a very similar philosophy. The whole group resulted primarily from the Reformation which taught "the responsibility of the individual to God both for his own life and that of others."[1] History has shown us that the Puritans had no conflicts with the concept of an established church, but merely with the particular established church in England. They wished to alter this religious system so that it would serve their purposes. When this became impossible, they revolted. These people were striving for independence, but not for freedom or toleration in religion. This is demonstrated by the fact that there was no religious toleration in the Puritan colony in

1. No. 1, page 67

New England later.

It is often remarked that there were many clergymen of high education and broad vision in the Puritan group. Recent historical works seem to refute this claim, showing that of the 281 Puritan clergymen in England about 1610, 176 had no university degree and only 31 had anything better than a master's degree.[1] The leaders were not among the intellectual elite of England, but they did succeed in eliminating some of the current social evils. A sombre mode of living was substituted for the overhearty enjoyment of life which characterized their predecessors.

The Puritan religious philosophy was steeped in Calvinism which taught that:

1. "God not only foresaw the fall of man, and the ruin of posterity in him, but arranged all by the determination of his own will."
2. "God chose certain individuals as his elect to be saved from all eternity by his gratuitous mercy, irrespective of human merit. The rest he condemned by a just and irreprehensible judgment."[2]

However happiness was achieved through this system, because the Puritans considered themselves the elect of God.

In New England, every effort was made to have the community life and government administrated by these same elect. They had no sympathy with the political independence of the individual and they condemned both vehemently.

The sum total of these attitudes may well be called

1. No. 1, page 73.
2. No. 1, page 170; page 149.

Puritanism. Coupled with absolute separation from European culture, it resulted in some stultification and lack of originality of thought.

ii.

The Schools in the Massachusetts Bay Colony

Because of the Puritans' strong belief in the individual's duty to God, it is natural they insisted upon the early establishment of schools. So it is natural that they should model their educational system upon the only one they knew, the English.

The earliest reference to a public school is to be found in the minutes of the town meeting at Boston on April 13, 1635, at which it "was generally agreed upon that brother Mr. Philemon Pormont should be entreated to become schoolmaster for the teaching and nurturing of children with us."[1] This was the beginning of the Boston Latin School which was a true Latin School, well exhibiting their limited curriculum of Latin and Greek. Not until well after the Revolution did they offer mathematics. Pupils were allowed to enter at the age of nine and enjoyed a concentrated course in the classics until they chose to leave or enter Harvard College. Soon after 1635 similar schools were founded in other parts of the state.

In 1636, John Harvard upon his death endowed Harvard College which was an added impetus to the Latin School.

The first movement toward compulsory education came with the law passed by the General Court in 1642, generally known as "Ye Olde Deluder Satan Acte", which provided that "the chosen men of the town are to see that parents train their

1. No. 12 Forward.

children in learning, labor and employment and, if not, upon presentment to the grand jury, or other information of their neglect, the said townsmen are subject to fine. They may impose fines upon such parents as refuse to give account of their children's education. With the consent of two magistrates they have the power to put for apprentice such children whose parents are not able and fit to bring them up."

The next legal provision of importance came in 1647 when the General Court decreed; "It is therefore ordered that every township in the jurisdiction, after the Lord hath increased them to the number of fifty householders, shall then forewith appoint one within their town to teach all such children as shall resort to him, whose wages shall be paid either by the parents or masters of such children, or by the inhabitants in general, by the way of supply, as the majorpart of those that order the prudential of the town shall appoint; provided those that send their children be not oppressed by paying more than they can have them taught in other towns; and it further ordered, that where any town shall increase to 100 families or house-holders, they shall set up a grammar school the master thereof being able to instruct the youth so far as they may be fitted for the university; provided that if any town neglect the performance hereof above one year that every such town shall pay five pounds to the next school till they shall perform the order."

This seems to imply a division between upper and lower schools, but the lower school failed to come into existence

1. No. 12. Forward.

until the latter part of the century. The first writing school in Boston was not formed until "November 1, 1684 at Prison Lane".[1] The traditional program of the writing school consisted of writing and arithmetic. Usually, these are the only subjects of instruction mentioned in the records. These schools were not completely primary in nature, as the entrance requirements said that the children must know how to read. Some representative dates of establishment are:

1. Dedham 1652
2. Dover 1658
3. Hingham 1670
4. Plymouth 1663
5. Watertown 1658

Another type of early colonial school was the Dame School. This was important only because it furnished the first opportunity for the education of girls. The rudiments of spelling, reading in the New England Primer, and the Cathechism were included in the course of instruction. Rarely was writing or arithmetic touched upon. The following description seems adequately to cover the usual situation: "The wife of Ebenezer Field, the smith, in 1790, taught a class of young children for twenty-two weeks of the warm season, and charged four cents a week. She educated her own children well; her own children well; her own daughter, Joanna, was the noted schoolmarm of the next generation. Mrs. Field was a woman of great energy and versatility. Besides keeping school, she made shirts for the Indians at eightpence each, made breeches for Ensign Field at one shilling sixpence per pair, and managed her own household with four young children."[2]

1. No. 12 Forward
2. No. 4, page 162

iii.

The Teachers in Massachusetts before 1780.

There are many evidences that in the Colonial Period there was a scarcity of schoolmasters. For instance, Governor Dudley writes to his son in England in 1642, "There is great want of schoolmasters hereabouts" [1.]

One of the reasons for this condition was the importance of the church in New England at this time. The church was dominant and the ministry was looked upon as the natural work of the college graduate. Little dignity was accorded the schoolmaster, and little encouragement.

Another reason for the few teachers was an economic one. His salary was low. Often he was paid in goods--wampum, bullets, cattle or even shoes serving as legal tender. Payment in silver varied from twenty to sixty pounds per annum.

Consequently, those who became schoolmasters were often unintelligent and shiftless. By far the best of them were the young college students or graduates who taught until they were able to enter a more desirable profession. One of these was John Adams who taught at the Grammar School in Worcester for three years. [2.]

The colonial teacher had many non-professional duties

1. No. 4, page 67.
2. No. 3, page 1 1.

such as digging graves and leading the Sunday choir. However, these divvi..lities were somewhat compensated by the fact that a competent teacher could hold his position as long as he wished. The classic example of this was Ezekiel Cheever who served for sevent years.

The following quotation describes a schoolmaster in Rosford about 1772, who has some of the characteristics mentioned above: "Browned by the sun and the heat while cultivating his arable acres, his hands like the sturdy yeoman rather than the schoolmaster, his gestures and walk betokening the commanding position which he holds....all are brought to our eyes while we hear him affirm that g-e-s spells 'guess'. Dignified in behavior, stern in aspect, harsh, often cruel in action, the schoolmaster typified the spirit of the age; although the colonies were often weary and heavy laden, they never wholly abandoned him as one of the pillars upon which their future depended."[1.]

iv.

The Beginnings of Mathematics in Massachusetts

As we have observed, mathematics did not enter into Massachusetts's first schools. There was a similar condition in England where mathematics was a desirable fine art but not a necessity. All attention was directed to Latin and Greek, so that the wisdom of the ancients and the Scriptures might be assimilated. Mathematics had little religious value, so that the

1. No. 14, page 9.

study of it was not encouraged.

When it did come into our curriculum, it seems to have come through popular demand, certainly not tradition. College entrance requirements did not mention it and neither did the laws of 1642 or 1647. We first hear of it in the various town records:-[1]

1. Dedham, 165[illegible]. "Schoolmaster agreed to teach to read English and the Accidence, and to write and the knowledge and art of arithmetic and the rules and practice thereof".
2. Boston, 1[illegible]4. Founding of the Prison Lane Writing School, where the traditional curriculum was writing and arithmetic.
3. Hingham, 1670. "He will teach and instruct in Latin, Greek, writing and mathematic".
4. Plymouth, 1674, ordered that "due attention be paid to writing and reading and arithmetic."
5. Haverhill, 1685. Schoolmaster "to endeavor to teach such as shall resort to him, as they shall desire, to read or write or cypher or all of them".
6. Sandwich, 16[illegible]. Schoolmaster "to teach all children sent to him to learn English and the Latin tongue, also writing and the art of arithmetic."
7. Muddy River (Brookline) 17[illegible]. Petitioned for a schoolmaster "to teach, to write and to cypher".
8. Sudbury, 170[illegible]. Schoolmaster to teach the children sent to him reading, writing and arithmetic."

From all these records, it is quite obvious that arithmetic

1. No. 14, page 260-62.

was not a compulsory subject and was considered a frill rather than a basic course.

The popular estimation of it can also be gathered when we note the cost of mathematics in comparison with the other subjects:

1. In Newbury in 1681, "readers free, Latin sixpence per week, writing and cyphering fourpence per week."
2. In Newton on 1761, "four pence per week per cyphering."
3. In Lynn in 1702, "two pence per week for reading, threepence per week for writing and cyphering and sixpence per week for Latin."[1]

From the above, we see that mathematics did not become immediately popular.

V.

Modes of Instruction in the Mathematics Class

During the years before our Revolution, the teacher either had no conscious mode of instruction or thought it unimportant, because he failed to write about. Our documentary evidence, except for the texts in vogue, is exceedingly slight.

We do know that materials were costly. Blackboards, of course, were unknown and slates were not to be introduced until many years later; usually sums were done on birch bark because paper was much too valuable for such a frivolous use.

Under these conditions, few pupils had books. Some tea-

1. No. 1-, page 26-.

[illegible]hers were without them. Consequently, the master was forced to dictate all the problems from the copy book which he had made as a pupil and now used for the classroom. The student worked the problems and possibly copied them into his cyphering book, if he were fortunate enough to own one. There was no examination of processes, no demonstration of principles. This is explained when we note that "he was considered an exceptional teacher who possessed a knowledge of fractions, and if some rare pupil managed to master fractions, too, he was judged a finished mathematician." Evidently the teacher hardly expected to make excursions into the rule of three brought neither quantity or quality to his instruction.

Perhaps the best first-hand description of the method was given years later (in 18[illegible]) by Warren Colburn when he addressed the National Education Association: "By the old system, the learner was presented with a rule which told him how to perform certain operations on figures and when these were done he would have the proper result. But no reasoning was given for a single step. His first application of the rule was on a set of numbers so large that he could not reason on them, had he been disposed to do so. And when he got through and obtained the result, he understood neither what it was nor the use of it. Neither was he sure that it was the proper result, being obtained more from magic than from reason."[3]

In the colleges, there were naturally different conditions

1. No. [illegible], page [illegible].
2. No. 1[illegible], page 1[illegible]5.
3. No. 14, page [illegible].

than in the primary or secondary schools. Until the early part of the eighteenth century, a course in arithmetic was the only mathematics in the college curriculum. To enter Harvard, the student was not required to know anything of the science of numbers, but "only so much Latin as was sufficient to understand Tully or like author, and make and speak true Latin in prose or verse and so much Greek as was included in declining perfectly the paradigms of the Greek nouns and verbs." We must also remember that mathematics was not studied until the senior year, arithmetic for three quarters and astronomy for the last quarter of the year. The importance attached to it may be inferred from the following table:

Subject	Time devoted weekly
Latin	10 hours
Philosophy	10 hours
Greek	7 hours
Rhetoric	6 hours
Oriental language	4 hours
Mathematics	2 hours

The theory or presentation of mathematics at Harvard College did not improve until the beginning of the eighteenth century. In 1726, the Hollis Professorship of mathematics was instituted at the college. Up to that time, the instruction was in the hands of tutors. This custom was maintained until about 1800, when there were only three professorships in the entire school.[1] Until 1726, almost any minister was considered competent to give the course in mathematics. They

1.No. 2. page 19.

held their positions for very short terms and the instruction was no doubt mediocre.

VI.

The Mathematics Texts

To those of us who believe New England to be the fountain of all knowledge and the source of all culture in the new world, it is a considerable shock to learn that the first mathematics books were published in Mexico. The University of Mexico was founded in 1550, Harvard in 1636. In 1556, Juan Diez Freyle published his Summario Compendsio in Mexico City, a book treating of gold and silver values with twenty-five pages on arithmetic and algebra. In 1623, Arte Menor de Arithmetica Practica was printed for Juan Ruys for Pedro Paz, the collector of tithes of the Metropolitan Church of Mexico. From all reports, this is accredited to be the first arithmetic published by an American.[1] The second treatise on mathematics (Arte de Arithmetica by Don A. Ra ton, the rector of Pasamonte in 1640) was from the same locality. There is no existent copy of this work.

In the North, as our schools were at first purely imitative, so were our texts directly imported from England. Probably one of the earliest was that of James Hodder, another was that of Cocker. Cocker's treatise is described as the first to be devoid of all demonstration and reasoning. This gives us some clue as to the character of later works, many of which

1. No. 9, page 349.

were modelled on Cocker. His text was widely known as we see from this exerpt from Benjamin Franklin's Autobiography: "....having one day been brought to the blush for my ignorance in the art of calculation, which I had twice failed to learn while at school, I took Cocker's treatise and went through it with the greatest ease."

The first arithmetical text written by an inhabitant of the future United States of America was the Arithmetick, Vulgar and Decimal, by Isaac Greenwood in 1729. Mr. Greenwood had the honor of being the first recipient of the Hollis Professorship at Harvard College. He was considered the most proficient professor educator in mathematics and natural philosophy in New England. Unfortunately, in 1738, he was relieved of his position because of having been "guilty of many acts of gross intemperance to the dishonor of God and great hurt and reproach of society."[1]

1. No. 2. page 24.

Greenwood's treatise deals with the following topics:-

Part I Chapter 1. Numeration.

Chapter 2. Addition.

Chapter 3. Subtraction.

Chapter 4. Multiplication.

Chapter 5. Division.

Chapter 6. Reduction.

Chapter 7. Vulgar Fractions.

Chapter 8. Decimal Fractions.

Chapter 9. Roots and Powers..

Chapter 10. Continued Proportion.

Chapter 11. Disjoint Proportion.

Chapter 12. Practice

Chapter 13. Rules Relating to Trade and Commerce.

Part II Chapter 1. Fellowship.

Chapter 2. Barter.

Chapter 3. Equation of Payment.

Chapter 4. Loss and Gain.

Chapter 5. Allegation.

Chapter 6. Position or the Rule of False.

Chapter 7. Interest.

Chapter 8. Rebate or Discount.

Chapter 9. Annuities.

Chapter 10. Of Reversion and Freehold Estates.

1. No. . page 24.

This arrangement is admirable for the eighteenth century and it was copied in later books. However, some of the later works displayed a different topical order which was decidedly inferior to the Greenwood method. Among these was The Schoolmaster's Assistant by Thomas Dilworth. The book was divided into three parts, the first on whole numbers, the second on vulgar fractions and the third on decimal fractions. The student was taught all the processes relating to whole numbers without realizing the existence of a fraction, either vulgar or decimal. Naturally, he felt, upon completing the book that there were three fundamental processes, whereas really there is only one.

The Dilworth method of approaching proportion is noteworthy:

> "Q. How many parts are there in the rule of three?
>
> A. Two; single and simple, and double and compound.
>
> Q. By what is the single rule of three known?
>
> A. By their terms, which are always given in the question, to find a fourth.
>
> Q. Are any of the terms given to be produced from one denomination to another?
>
> A. If any of the terms are in the several denominations, they must be reduced into the lowest denomination mentioned."[1]

How a pupil could have understood this when he was being introduced to proportion for the first time is a mystery. The Dilworth definition of a fraction is interesting, too: "A fraction is a proper number and signifies the part or parts of a whole number."[2]

1. No. 4 page 44.
2. No. 4 page iii.

There are several characteristics of the early book which may be used to summarize the books of the entire period:

1. Fine unattractive print.
2. An abundance of disconnected rules.
3. No demonstration.
4. An abundance of confusing technical language.
5. Inaccurate expression, "feet multiplied by feet give feet".
6. No attempt to build upon the past experience of the learner.

At Harvard College, <u>John Ward's</u> <u>Introduction to Arithmetic</u> was used. This text had many of Dilworth's faults. Arithmetic, Algebra, Geometry, Conick Sections, and Infinities were discussed. The part on geometry consists of definitions, twenty proven problems and twenty-four theorems. They are presented entirely without rigor and show a lack of Eucidean method.

Having considered the paucity of schools, mathematical theory, teaching and texts, the reader will realize that in the colonial period mathematics was of no major importance or interest.

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THE TRANSITION PERIOD

1776--1820

Crucial Years and Their Effect Upon Education.

The War of Independence had the usual effects of any war, but somehow, when read of it, it seems that the colonists suffered to a particularly great degree. For the first time, they found themselves completely isolated. What food they were to have could not be imported. They were forced to supply themselves with all the necessities of life and what few luxuries existed at that time. The army had to be maintained at any cost, and our struggling infant industries had to be fostered. In any such emergency, a peoples' first consideration is their own condition. This was certainly true all through the Revolution. Education seemed unimportant and could not be emphasised. Most of the rural and parochial schools were closed, because the young sons were needed on the farms. In the areas of English occupation, the pursuit of learning was necessarily prohibited. Even in the larger cities, there were only a few educational institutions maintained throughout the period of the war. The circumstances were even more adverse as far as the colleges were concerned. Both students and professors were at the front, while the dormitories were used as barracks. Education was forgotten because there were so many more important things to do than to go to school.

However with the formation of our national government, an altogether new spirit permeated the country. It was an ambitious, confident feeling that spread into every branch of our national life. It was so strong that many writers have been prompted to say that more real progress took place directly after the war than did one hundred and fifty years preceeding it. We began to feel that progress in education was necessary so that we would in no way be inferior to the English. American texts came pouring off our presses and there was genuine effort to counteract all reactionary principles. However, in most cases, the result was merely an elaboration of old ideas. There were no outstanding examples of scholarship.

Probably, the most important general change was the founding of the academy. In the Colonial period, we have seen that the two most popular types of schools were the Latin and the writing school. For those who wished to enter Harvard College, the Latin School was admirable. For those who wished to learn merely the rudiments, the writing school served the purpose. For the ordinary boy, who did intend to enter a profession but wished to follow a business career, there was no means of education.

As early as 1761, there was an expression of the need for a new type of secondary school. Lieutenant-Governor Dummer bequeathed his property for the use of a grammar school. Being in private hands and beyond the control of local authorities, this school soon adopted the curriculum of the academy. Not until twenty years later was the first academy actually founded by Samuel Phillips at Andover, the Phillips Andover Academy. In 1797, the legislature of Massachusetts recognized the academy as a legitimate part of the Massachusetts school system and made provision of public lands for their support.[1] Coming at a time when the only Latin schools worthy of the name were in a few large cities, this was a virtual admission that the age of their popular demand was past.

The usual curricula of academies can be inferred from that of Phillips Andover in 1789:

> "English, French, Latin and Greek languages; geography, arithmetic, and practical geometry; rhetoric, history, natural and moral philosophy, logic, astronomy and natural law."[2]

and that of Woburn Academy in 1815:

> "It is our humble and pleasing duty to instruct young lads in the regular and genteel behaviour, and in the various branches of literature, viz; reading, writing, and arithmetic, geography, bookkeeping, English, grammar, rhetoric, composition, and the Latin and Greek languages. Likewise, Astronomy, navigation, or surveying will be taught to such as may be inclined to acquire a knowledge in either of these branches."[3]

The later town academy fulfilled these three needs:

1. Educational facilities for those who wished to fit themselves for a non-professional career.
2. A new body of subject matter of a more general character than that used in the Latin School.

1. No. 6, page 32.
2. No. 6, page 33.
3. No. 6, page 33.

3. Higher education for girls.[1]

ii.

The Teacher of the Transition Period

As in the colonial period, our information depends upon very slight documentary evidence, the large part of it being inferred from old texts.

Upon examination of extand evidence, one finds:

"Joseph T. Buckingham tells us how in 1790 or 1791 when he was about twelve years old, he began to learn arithmetic, 'I told the master I wanted to learn to cypher. He set me a sum in simple addition, five columns of figures in each column. All the introduction he gave me was, "add the figures in the first column, carry one for every ten, and set the overplus down under the column". I supposed he meant by the first column the left hand column, but what he meant by carrying one for every ten was as much a mystery as Simpson's riddle was to the Phillistines. I worried my brains

1. No. 9, page 150.

an hour and showed my figures t the master. You may judge what the amount was when the columns were added from left to right. The master frowned and repeated this for instruction: "add up the column on the right, carry one for every ten, and set down the remainder". Two or three afternoons were spent in this way, when I begged to be excused from learning to cypher, and the old gentleman with whom I lived thought it was time wasted. If I attended the school any further at that time, reading, writing and a little spelling were all that was taught."[1]

Mr. William B. Fowle gives an interesting account of the method of John Tileston, who was chief writing master in a reading school in Boston about 1790, "He loved routine. The custom was for the master to write a problem or two in the manuscript of the pupil every other day (printed arithmetics were not used in the Boston schools until the writer left them). No boy was allowed to cipher until he was eleven years old and writing and ciphering were never performed on the same day. Master Tileston had thus been taught by Master Procter and all the sums he set for his pupils were copied exactly from his old manuscripts. Anybody could copy the work from more advanced manuscripts but the principles of arithmetic were never explained to the copier while he was at school. Indeed the pupils believed the master could not do the sums he set for them."[2]

Deacon Joseph Hawes says his method about 1820 was: "Those

1. No. 3, page 5?.
2. No.12, page 49.

in arithmetic having books of different authors got their own sums, wrote off their own rules, etc. If they wanted to make inquiries concerning questions, I would direct them to stand up and read their question, and if the scholar next him could show him, I would request him to; if not, if I had extra time, I would tell him the rule by which the question was to be done. If he then met with difficulty, I directed him to take it home and study late at night and have his answer in the morning."[1]

Evidently the calibre of teachers had improved very little. Not only were some of them very badly informed concerning the subject matter, but their moral standards were sometimes poor. The tales of drinking and gambling masters are many and remind us of the earlier conduct of Isaac Greenwood.

At Harvard, the instruction was on a much higher level than in the secondary school. In 1803, arithmetic was made a prerequisite to college. However, arithmetic was still taught as a college subject with Euclid added in the sophomore year. The first mention to be found of the introduction of algebra was in 1766, when a few ambitious seniors were writing theses in the subject. These were usually a series of solved problems put into the finest manuscript form. The work was very elementary, of the type which is done now by every good secondary school student.[2]

1. No. 20, page 68.
2. No. 16, page 42.

iii.

The Texts of the Transition Period.

Of the mathematics texts in use at this time, undoubtedly the most popular was that of Nathaniel Pike, who published the first edition of his New and Complete System of Arithmetic in 1788. Pike was a graduate of Harvard College and later a public official at Newburyport, Massachusetts. It is interesting to note in the preface that Mr. Pike's book garnered many enthusiastic recommendations. For example, Ezra Stiles, president of Yale College said: "Upon examining Mr. Pike's system of arithmetic in manuscript, I find it to be the work of such mathematical ingenuity, that I esteem myself honored in joining with the Reverend President Willard and other learned gentlemen, in recommending it to the public as a production of genius, as a book to be taught in schools, of utility to the merchant and well adapted to university instruction. I consider it of such merit that it will probably gain a very general reception throughout the republic."[1.]

Because it was the text chiefly used between 1875 and 1825, the writer chose to discuss Pike somewhat in detail. The first topic in Pike's book is numeration. This requires the committing to memory of the entire decimal counting system with such illuminating remarks as this: "A cypher though it is of no value itself, yet it possesses a place, and when set on the right hand of figures in whole numbers, increase their value in the same tenfold proportion; thus 9 signifies one nine, but if a cypher is placed on its right hand thus--90, then 90 becomes ninety."[2.]

1. No. 15, Forward.
2. No. 15, page 2.

The counting proceeded through the millions to sextillions and duodecillions. In the next section, the student is introduced to addition, where he is told to follow the rule, to check his work and, at all times, to use the addition table in his computations. The same idea is reiterated throughout the four fundamental processes, when he presents exercises which demonstrate every general rule and every deviation from them. For instance we have:

1. What number must we multiply by 9, so that the product may be 675? (The method and answer is generously supplied, 675 ÷ 9 = 75).
2. What is the difference between six dozen dozen and half a dozen dozen? What is their product and the quotient of the greater by the less?[1]

These problems would certainly not appeal to the ordinary child.

There follows the inconceivably extended tables of weights and measurements. The important list of monetary denominations preceded troy, avordupois, apothecaries, weight and cloth, long, time, land, solid and ale measure. One can appreciate their potential difficulties when we read this:

"A comparison of the American Foot with the Feet of Other Countries. The American Foot being divided into 1000 parts, or into twelve inches, the feet of several other countries will be as follows:

1. No. 15, page 62.

	Parts	Inches	In.	Points
America	1000	12	0	0
London	1000	12	0	0
Antwerp	946	11	4	132
Bologna	1204	14	5	225
Bremen	964	11	6	4[illegible] [1]

In the section on fractions we find a remark in the corner of one page which is of interest, although Pike thought it of little importance and the idea was not elaborated until forty years later:

"If the denominator of any number of a compound fraction be equal to the numerator of another member thereof, these equal numerators and denominators may be expunged and the other members continually multiplied, will produce the fraction in lower terms."[2]

Decimal fractions are discussed, but only in connection with the new Federal Money. In the same superficial vein, there are articles on Tare and Trett, Fellowship, and the Rule of Three. In spite of the acceptance of Pike, the writer has observed many of the faults of a bad text--elaboration of difficulties, emphasis of the copy book method and ambiguity.

An arithmetic which rivalled Pike's in general usage was Nath and Daboll's <u>Schoolmaster's Assistant</u>. There are a great many similarities between this and Pike, but it did have the following improvements:

1. Introduction of Federal money immediately after the addition of whole numbers.

2. Demonstration of process of how to find the value

1. No. 15, page 72.
2. No. 1[illegible], page 163.

of goods immediately after simple multiplication.
3. Decimal fractions preceded vulgar fractions.[1]

The third outstanding text was the Scholars Arithmetic by Daniel Adams. Mr. Adams was graduated from Dartmouth in 1797 and later became teacher, physician and editor. In the third edition of his work he tells us that his book has been a success because it "has relieved masters of the heavy burden of writing out rules and questions under which they have so long laboured to the manifest neglect of other parts of their schools."[2] He adds, "To answer the several intentions of this work, it will be necessary that it should be put into the hands of every arithmetician. The blanks after each example are designed for the operations by the scholar, which being first wrought upon a slate or waste paper, he may afterwards transcribe into his book."[3] Here Mr. Adams states his faith in the complete efficacy of the copy book method. After the student has worked all the problems in his text, he supposedly keeps it as a desirable and necessary guide for all his future work. The author is optomistically looking forward to the time when every individual student would have a text.

The following books are of lesser importance, but have some noteworthy passages, quite characteristic of the times:

In 1792, John Vinall published his <u>System of Arithmetic</u>. Here we have the first mention as far as the writer has been

1. No. 24. Forward.
2. No. 23. Forward.
3. No. 14. Forward.

able to determine, of the idea that every pupil should have a text. He says: "In our large schools of 300 boys, it is impracticable for the masters to attend to them in a manner that will keep them employed the whole time they are in school. Consequently, idleness will creep in among them, which will produce mischief. In publishing the above treatise, my intention was that each in my first class should be furnished with one of the books, and, at the leisure time mentioned, he should be obliged to study arithmetic which would prevent idleness and at the same time improve his mind".[1]

In 1802, a Concise Introduction to Practical Arithmetic by Samuel Temple was put on the market. It had a very simple purpose which was quite in the spirit of the times: "The principal design in publishing this work was to furnish schools in the United States with suitable rules for calculating Federal money".[2]

Phineas Merrill in his Scholars Guide to Arithmetic says: "When a scholar begins a rule, the method I take is this: first, I explain the nature of its use in business, bringing natural occurances to convey my ideas. Then I take a question in the rule and work at large, explaining how it applies the rule. My scholars will remember an expression I often use, 'Compute by rule, not by guess'."[3] Here we have the glorification of the rule method not only in practice, but in theory.

In William Kinne's A Short System to Practical Arithmetic, we have a beautiful expression of the usual attitude of the

1. No. 24. Forward.
2. No. 25. Forward.
3. No. 14 Forward.

bookmaker. "Perspicuity and conciseness are peculiarly requisite in all the works intended for instruction, but in none more so than in that of arithmetic. With a view to remove the obstacles which oppose the progress of the learner and the labors of the instructor, all remarks and demonstrations are thrown into the form of notes".[1]

Next we have two important attempts to make texts which would be appropriate for the use of children. The first is Arithmetic Made Easy to Children by Emon Kimber. Unfortunately, the book does not live up to its title, but is merely more concise than the usual arithmetic. In 1813, we have a more successful attempt. Dudley Leavitt in his Elements of Arithmetic really strives for simplicity. He says, "Those who have written small arithmetics, although their professed object was to instruct youth, either mistaking the talents of youth or wishing to display their profound erudition, have interlarded their performances with such a multitude of difficult and improper questions that it would require a conjurer to decypher its meaning. In the following compendium, the author has endeavored to accomodate the rules and examples to the interests of the learner".[2] There are a few simple examples and this is the first hint of a new era to come.

1. No. 12, Forward
2. No. 13, Forward

From this rapid survey we can safely enumerate the following characteristics of the transition text:

1. A conglomeration of rules covering every possible case--all meant to be memorized by the learner.
2. Total absence of demonstration.
3. Teaching all the fundamental processes as rules.
4. Emphasis of the copy book method.

In spite of the increased supply of American texts, most of the schools had an inadequate supply of books.

In Harvard College the most popular book was Thomas Simpson's Elements of Euclid and his Treatise on Algebra. According to Cajori: "All demonstrations are here given by themselves in the manner of notes placed below a horizontal line in the page. These could be taken or omitted by teacher or pupil at pleasure and were generally omitted. The explanations lacked simplicity and we need not wonder if they were looked upon by some as rather tending to throw new difficulties in the way of the learner than to the facilitating of his progress."[1]

In the Legendre which was also in use during the transition period at Harvard, one notices that:

1. Demonstration are of a complex character.
2. The content is of the present secondary school grade.

1. No. 3, page 26.

3. The student is not required to do any original problems.
4. The author attempts to prove every statement in an overrigorous fashion.

During these years after the Revolution, the writer has noted an added recognition of mathematics in the curriculum. This was the cause of more texts and consciousness of teaching method. However, it did not result in any major pedagogical attainments. There was simply a rehashing of the poor teaching method which New England inherited two hundred years earlier from England.

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PESTALOZZI AND HIS INFLUENCE

I.

The Massachusetts School System (1820-1830)

An interesting commentary on the condition of the schools in Massachusetts about 1824 was made by James Gordon Carter. He said: "The decline of popular education among us, or rather the comparative retrograde motion of the principal means of it, has been more perceptible during the last twenty or thirty years than it ever was at any previous period. And, in the meantime, there has sprung up another class of schools, more respectable indeed in their character and better answering the demands of a portion of the public, but not free. The academies are public but not free. The conditions of attendance exclude nineteenth twentieths of the people from participating in the advantages which they are designed to afford. However, better schools and better instruction are demanded than the academy in its present state can afford and these must be supplied."[1.]

In this manner Carter points out the causes of the educational difficulties of the period. The demands which he voiced were to be repeated and to result in that very interesting development of a new institution, the high school. Boston was the center of experimentation with its English High School founded in 1821. In 1825 there was established:

1. No. 5, page 5.

"The Monitorial School for Girls

Public notice is hereby given that, on Tuesday, 15th, next, the School Committee will meet for the appointment of a Master for the High School for Girls, about to be established in this city. The school is to be conducted upon the system of monitorial or mutual instruction and it is to be expected that the master will be prepared to teach, on this system, so far as it will be practicable: Reading, Writing, Spelling, words and sentences from dictation, English Grammar, Composition, Modern and Ancient Geography, Intellectual Arithmetic, Rhetoric, General History, History of the United States, of England, Rome and Greece, Bookkeeping by Single Entry, Elements of Geometry, Demonstrative Geometry, Algebra, the Latin and French Languages, Natural Philosophy, Chemistry, Botany, Logic, Astronomy, the use of the Globes, Projection of Maps, Principles of Perspective, Moral Philosophy, and the Evidences of Christianity."[1]

There followed attempts in other towns throughout the state to establish high schools, but it should be emphasized that these were purely individual in purpose and effect. The first official state mention of the high school came with the law of 1827:

"Be it enacted by the Senate and House of Representatives in the General Court assembled and by the authority of the same.

1. No. 13, page 45.

That each town or district within this Commonwealth, containing fifty families or house holders, shall be provided with a teacher or teachers, of good morals, to instruct children in orthography, reading, writing, English Grammar, Geography, Arithmetic, and good behavior.......And every city containing five hundred families or householders shall also be provided with a master of good morals competent to instruct, in addition to the branches of learning aforesaid, the history of the United States, Bookkeeping by single entry, geometry, surveying and algebra; and in every city of four thousand inhabitants, such masters shall be competent, in addition to the foregoing branches, to instruct the Latin and Greek languages, history, rhetoric and logic."[1]

This law dealt a death blow to the Latin School by offering many of its advantages and none of its disadvantages. However, the growth of the high school was slow. There ensued a long struggle against adverse circumstances because of the district system. The effects that this system had upon the early high school are worth mentioning:

1. The division of authority caused opposition between elementary schools and high schools.
2. Lack of uniformity in elementary schools handicapped the high school.
3. Lack of grading in the elementary schools prevented proper preparation for the work of the high school.
4. The efforts of districts to have their own high schools postponed the coming of the general town school.

1. No. 16, page 17.

ii.

Pestalozzi

While the United States (Massachusetts in particular) was bending its efforts toward a proper educational organization there were new ideas arising in the field of pedagogy. It would be pleasant to relate that these originated in America, but this was not true. Like many other novelties these new conceptions came from France. There, directly after the French Revolution, there was a new impetus toward popular learning. The pedagogy which had the greatest influence in America was that of Heinrich Pestalozzi. From a psychological point of view, Pestalozzi is famous for his emphasis of sense perception. Here, says, is foundation of all knowledge. He insisted that although there were five ways of arriving at truth, (chance, surroundings, personal will, vocation and anology) all of these were based on sense perception. He remarks: "The art of facilitating the clear consciousness of single objects from sense perception to accurate thought and judgment on them, by naturally organized means of instruction, which have been arranged in psychological gradation, is in anything but a high degree of perfection and thoroughness in our hands. Instead of carefully striving to group and separate correctly the objects perceived by the senses, people want more and more to teach children to think by extension of the number of objects presented for reflection or by the teaching of logic."[1] With this idea as a basis, Pestalozzi's educational

1. No. 26, page 291.

method is this:

1. Intensive and not extensive education.
2. Language as the basis of instruction.
3. Guiding ideas or rubrics when a new concept is being introduced to the mind of the learner.
4. Simplification of the mechanism of instruction.
5. Popularization of science.
6. Mutual or class instruction.

All these methods were unknown to the American schoolmaster of 18 0.

Mathematics or the science of form and number, Pestalozzi sums up in what he calls the ABC of sense perception. He emphasizes the futility of knowledge gained by the mere use of memory: "If we commit to memory the fact that 3 + 4 = 7, the inner truth of this seven is not known to us."[1] Quite naturally, Pestalozzi begins his teaching of arithmetic by dealing with the numbers one to ten by means of objects or lines or dots. After the child finds himself very well versed in counting these ten numbers, he would pass on to figures. Pestalozzi felt that this method was noticeably better than the old English method, because learning became the basis of clear conceptions, and because learning, as rooted in sense perceptions, became distinctly easier to the child. Thus the learner is introduced to form as early as number and the ABC of sense perception becomes the linking of these two fundamental aspects of quantity in a pedagogy rooted in a primitive sensation psychology.

1. No. 26, page 255.

iii.

Warren Colburn

After two hundred years of continued practice, any idea even when a better one is offered to society, is hard to dislodge. Habit and tradition are probably as strong as any of our human means of determining conduct. Consequently, it was a matter of importance when Warren Colburn adopted Pestalozzian ideas with such alacrity. Colburn graduated from Harvard in 1820, spent the following years writing and teaching and then turned his attention to industrial administration. However, he is chiefly remembered for his educational endeavors. It is undeniably true that he is the first American educator to realize the potentialities of oral instruction, object teaching and mental arithmetic. In an address before the National Education Association, he gives an interesting explanation of his "new system": "By the new system, the learner commences with practical examples in which the numbers are so small that he can easily reason on them. He is compelled to exert his own powers, and every new example increases his confidence in these powers. The scholar makes his own rules. The method should be this:

1. Teach one thing at a time.
2. Be careful in your selection to choose the easiest first, then the next harder, etc.
3. The learner should never be told directly how to

do a process.

4. All illustrations should be given by practical examples having reference to sensible objects.
5. First, it is necessary that the teacher should be able to trace readily the associations of his pupils, when he hears them in recitation.
6. The scholar should be allowed to reason his own way.
7. Make the students study.
8. Do not hear a recitation unless it is well prepared, otherwise there will be inattentiveness."[1]

The first tangible evidence of the new method came with the publication of Colburn's First Lessons in Arithmetic in 1822. From the preface of this book, we read: "As soon as children have the idea of more or less, and the names of the first few numbers, they are able to make small calculations. And this we see them do every day about their playthings. The fondness which children usually manifest for these exercises and the facility with which they perform them, seem to indicate that the science of numbers, to a certain extent, should be among the first lessons taught to them. To succeed in this, however, it is necessary rather to furnish occasions for them to exercise their own skill in performing examples, rather than to give them rules....The idea of number is first acquired by observing sensible objects. We make calculations with regard

1. No. 17, page 421.

to those objects; soon we see that these calculations will apply to objects very disimilar and finally that they can be made without reference to any particular thing....Examples of any kind upon abstract numbers are of very little use, until the learner has discovered the principle from practical examples."[1]

Ably following his theory, Colburn starts his treatise with: "How many fingers have you on your right hand? How many on your left?" In a definitely philosophical manner, Colburn treats each of the traditional arithmetic topics first practically, then theoretically. It is somewhat amazing to consider that over three million copies of the book were sold. Here again we are apt to evaluate in terms of our own generation, but we must remember that books were hard to get, transportation was slow.

In spite of this, Colburn sold this huge aggregate of copies.

1. No. 8, Preface.

iv.

Some Followers of Colburn and Some Opponents.

In 1822, John Lyman Newell published his New American Arithmetic. Mr. Newell cannot claim to be Pestalozzian in tone, but he does strive for the simplified method when he precedes each new explanation with a list of preparatory oral questions. Here the improvement stops.

Victor Value in his Arithmetic, Theoretical and Practical does an excellent job and gives these suggestions: "Of all the branches of learning, arithmetic is the only one taught without explanations. Its fundamental principles are totally disregarded and why? Are these looked upon as self evident? Daily experience plainly shows that they are not self evident, for they are but little understood. Conditions irrestibly lead us to conclude that it would be advantageous to teach arithmetic theoretically as well as practically. The advantages are so obvious that it is a matter of astonishment that the attempt has not been made before."[1] He continues by hinting of advantages to be had in class instruction and oral expression on the part of the student. Value was a very progressive gentleman.

The Pupils Arithmetic by Seth Davis also shows up-to-date tendencies. He remarks: "There are no answers given to any of the questions, as the long experience of the author has fully convinced him that it is attended with manifest disadvantage to the pupil in acquiring the practical use of numbers. When

1. No. 25, Preface.

any question is involved in so much obscurity as not to be clearly understood, the manner of solving it is directly communicated by means of other questions immediately preceding it."[1]

Mr. Boswell Smith in his Practical and Mental Arithmetic professes to be devout disciple of Pestalozzi: "In this work, the author has endeavored to make every part conform to this maxim......'objects should proceed ideas'. A child's seeming stupidity in learning arithmetic may, perhaps, be a proof of intelligence and good sense. It is easy to make a boy, who does not reason, repeat by rote any technical rules a writing master lays down for him, but a child who reasons will not be thus easily managed; he stops, frowns, hesitates, questions, is wretched and refractory. This is taken into consideration in the following treatise."[2]

In the academy and higher schools, Theodore Walker's Elements of Geometry was popular. Here, although the change was not as complete as in the arithmetics, we read that Euler and Legendre are generally studied throughout New England, but that he considers them too hard for secondary schools. He therefore omits propositions which are not necessary for the understanding of subsequent work, and definitely simplifies the general wording of the text. For instance:

Walker: A line is the path described by the motion of a point.

Legendre: A line is the shortest way from one point to another.

Walker: Two straight lines are parallel when, all

1. No. 10 Preface.
2. No. 22 Preface.

the perpendiculars let fall from points in one to the other are equal.

Legendre: Two lines are said to be parallel when, being situated in the same plane and produced ever so far both ways, they do not meet.

In 1830, Bernard's Elementary Arithmetic came out and should be announced with trumpets and song. This is the first book to the writer's knowledge which used pictures as an aid to beginners. These are crude drawings but counting horses and chariots would undoubtedly be more fascinating to young people than counting dots.

With a keen appreciation of the trend of the times and more than good business sense, Oliver Shaw, in 1832, manufactured the visible numerator and published a booklet about it. This numerator was merely a mahogany box filled with different sized cubes so that the children could be reached through the eye "in accordance with the philosophy of Locke, Reid, Stewart, Byron and others". Mr. Shaw was undoubtedly our first manufacturer of teaching aids.

In Peter Parley's Method of Teaching Arithmetic to Children, we have a text created for the use of the primary grades. Says Parley: "I have here attempted to write a book of arithmetic which shall prove as amusing to children as a book of stories." [1] He has succeeded. His book of contents reads in part:

Chapter 5. About Tables.

Chapter 6. About Dogs.

Chapter 7. About Horns

Chapter 8. About Soldiers.

1. No. 19 Preface

Each chapter has a picture preceeding it and the questions are based upon these pictures. Undoubtedly, Parley was considered radical by his contemporaries, but he may be regarded as a man of genius.

The authors which have been listed above were the most progressive of the era. They were not in line with society or practice in general. Although delighted to greet their new ideas, we must realize that the majority opposed them. Many of the schoolmasters neither agreed or disagreed. They simply had never heard of Pestalozzi. This is shown by the persistence with which texts rolled off the press and incorporated all the old Pike, Daboll and Adams precepts. Among these were Day's Algebra, Leduck Thompson's Youth's Assistant, Ruter's Juvenile Arithmetic and Frederick Emerson's North American Arithmetic. Unfortunately these texts did not meet with disfavor, but had a wide circulation. Hassler remarks in his Elements of Arithmetic, that he can "promise the reader more satisfaction from his text than if he were to pass the time in a public house or barroom and lay out the cost of pencils and slate in vile liquer."[1] Considering the contents of this treatise, the writer is inclined to disagree with him.

1. No. 15 Preface.

V.

Elementary Mathematics at Harvard.

Since 1816, Arithmetic had been required for entrance to Harvard. In 181[illegible], some Algebra was added, with more in 1825. Gradually, all elementary mathematics was being forced into the curriculum of the academy and high school. In the freshman texts, where the elementary mathematics remained, we notice a fondness for French mathematicians, a general trend away from Euclid and the embellishment of many books with fine diagrams.

The influence of Pestalozzi was being felt even in the colleges, because here, too, the mind was being appealed to through the eye. However, the more elegant and advanced French investigators were claiming the attention of the students, while arithmetic, elementary algebra and plane geometry were being tried by their younger sisters and brothers in academy and high school.

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HORACE MANN AND HIS INFLUENCE

I.

Horace Mann, His Work

In 1837, there arose to prominence one of America's greatest educators, Horace Mann. He was appointed Secretary of the State Board of Education, an office which he himself was instrumental in creating. Of course, Mann was years ahead of his time. He was a gentleman of unusual ability. As Secretary of the Board of Education, he had merely persuasive power, but accomplished much. On one hand he made magnificent efforts toward improving school organization. He lauded the idea of state centralization and condemned that of the district system. On the other hand, he urged higher pay and better preparation of teachers. By 1847, normal schools were founded in West Barre, Bridgewater and Westfield.

The school children were probably the first concern of Mann. For their special benefit, he urged more physical education, better schoolhouses, school libraries, graded schools and classroom apparatus.

To promote accuracy and efficiency, Mann placed school registers in the hands of teachers, and reported on their results with statistical accuracy.

To further the dissemination of educational information, he edited and circulated the Common School Journal.

Naturally, all this added activity had effects both direct and indirect on the actual teaching of mathematics in the classroom.

II.

Horace Mann, His Views and Actions as to Mathematics.

In his famous reports as Secretary of the Board of Education, as early as 1840, Mann began expressing views which had an important effect upon mathematical instruction in the years that followed. For instance, he said: "Females are beginning to be employed, to a considerable extent, in the winter schools. Certainly, they are infinitely more fit than males to be the guides and exemplars of young children."[1]

According to Mann an ideal teacher is one who has:

"1. Knowledge of common school studies.

2. Aptness to teach.

3. Power to manage, govern and discipline a school.

4. Fitness to teach good behavior.

5. High moral character."[2]

In 1841, he remarked: "In some schools, an arithmetical key is in constant use by means of which the pupil always knows the number of places of figures, and the value of them at which he is to aim; and this knowledge becomes one of the elements in the calculating process by which the problem is to be worked. In this case, arithmetic degenerates into the art of obtaining, from known

1. No. 1, page 46, 1840.
2. No. 1, page 4[illegible], 1841.

data, on unknown principles, a known result whether right or wrong; instead of being that perfect science, which proceeding from known data, on known principles, evolves the true but unknown result, with infallible certainty."[1] He adds, "Almost every week, if not every day, the young arithmetician, in solving his imaginary questions, disposes of such quantities of goods as would make or ruin the fortune of a wholesale dealer; he makes calculations respecting such sums of money as but few capitalists have the disposal of. Such figures do not give them a feeling of reality and I doubt if he will meet them later. If he were led to imagine the schoolhouse a warehouse and individuals in the class as agents or owners by whom the business was transacted, he would have a new sense of responsibility and interest."[2]

Mann's admiration for foreign educational systems is unbounded. That it is the source of his many new ideas we can see by these excerpts: "In the public schools of Holland, large sheets were hung upon the walls of the room, containing facsimilies of the current coins of the kingdom."[3] As to the Scotch schools, "if a teacher cannot excite the attention of his class, he is pronounced without further inquiry incompetent to teach."[4] In the German Schools: "Thus with frequent reference to the blocks to keep up attention by presenting objects to the eye, the simple numbers were handled and transposed in a great variety of ways. In this lesson, it is obvious that counting, numeration, addition subtraction, multiplication and division were all included,

1. No1. page 53, 1841.
2. No1. page 55, 1841.
3. No. 1, page 87, 1842.
4. No. 1. page 8 , 1842.

yet there was no abstract rule or unintelligible form of words to be committed to memory. Nay, these little children took the first steps in the mensuration of superficial and solids, by comparing the length and contents of one block with those of others.[1]...."It struck me that the main difference between their mode of teaching arithmetic and ours consist in:

1. Their beginning earlier.
2. Continuing the practice in the elements much longer.
3. Requiring a more thorough analysis of all questions.
4. Not separating the processes or rules, so much as we do from each other.
5. Proceeding less by the rule and more from the understanding of the subject.

...... In algebra and geometry classes, I unvariably saw the teacher drawing diagrams and explaining all the relation between their several parts, while the pupils copied the figures and took down brief heads of their solution; at the next recitation they were required to go to the blackboard, draw the figures and solve the problems themselves. How different this mode of hearing a lesson from that of holding the text in the left hand while the forefinger of the right carefully follows the printed demonstration under penalty, should the place be lost, of being obliged to recommence the solution."[2]

From the foregoing, we recognize that Mr. Mann has set up these principles:

1. Women are excellent teachers for young people.
2. Every teacher must have certain native qualifications

1. No. 1, page 103, 1843.
2. No. 1, page 104, 1843.

and prerequisite training.

3. Every teacher must recognize his responsibility of holding the attention of his class.
4. Object teaching is highly desirable.
5. Juggling large figures, unless couched in realistic language, should have no part in our curriculum.
6. The teaching of the fundamentals should begin early.
7. The fundamentals should be learned through drill not through memorization.

These ideas may sound commonplace, but they were considered educational fads at this time.

In 1845, Mann coupled his words with actions and impressed upon Boston schoolmasters the desirability of an annual inspection, not cursory as had formerly been the case, but a thorough one in the form of a written examination. Here is our first American survey. The committee showed unusual wisdom in preparing and administering the tests. The questions were well graded and presented under uniform conditions, collusion was avoided and impartiality reigned. The results were just as Mann had anticipated: "In arithmetic, the possible number of right answers was 3080, the actual number was 107-. In considering the results, it must be remembered that not all the pupils in the first division were examined, only those whom the master chose to present for examination. For instance, out of 1215 scholars in the first class in the writing schools, only a carefully selected 268 took the arithmetic tests. These selected pupils made a score of .5% in accuracy."[1]

1. No. , page 52.

Now Mann had cause to remark: "What a pile of thunderbolts. If the masters see fit to assail me again, I think I can answer them in such a way as to make it resound to the glory of God."[1] Whether he realized it or not, Mann was now in a very commanding position. Henceforth, he would have state-wide influence, teachers superintendents, and school committees would lend an appreciative ear to his suggestions. From then on, his efforts increased and his remarks about mathematics began to be heard throughout the state.

However much we may approve of Mann's stand, we can fully appreciate it only when we turn to his own Arithmetic, Practical and Applied, in which he says: "Instead of groping along the mole-path of an irksome routine, with little other change than from dollars and cents, to pounds and pences, or some other familiar currency, and with little other variety than from cloth to corn, or some other commonplace commodity, it derives its examples from biography, geography, chronology and history; from educational, financial, commercial and civil statistics; from the laws of light and electricty, of sound and motion, of chemistry, and astronomy, and other of the exact sciences."[2] His admirable text deals shortly with fundamentals, (as this not meant to be used in the primary grades) and then progresses to chapters on The Farm, The Garden, The Household, Strength of Materials, The Road, The Engineer, The Laboratory, The Counting House, Statistics. Evidently, not only must we have a text which is adapted to the spirit of the times, but we must also have trained teachers and enlightened pedagogy. No wonder Mann is called our first American educator.

1. No. [illegible], page 7.
2. No. 24, Preface.

iii.

After Horace Mann, the Texts.

The writer wishes to emphasize particularly that Horace Mann merely introduced a new era. He did not remain to see that it was properly inducted into practice. He was heard, but not wholly heeded. Progress was being made by the intellectuals, but this progress was not yet ready for popular consumption. That the old text was still desired is demonstrated by the fact that new editions of Davies's <u>Practical Arithmetic</u>, Emerson's <u>North American Arithmetic</u>, Russell's <u>National Arithmetic</u> were put on the market and enjoyed a large sale.

After 1860, we have the publication of many noteworthy books. It must be remembered that the texts represent the most advanced theory of the time and that practice in the classroom is always years behind this pedagogy.

In arithmetic we have Burnham's <u>New Edition of Arithmetic</u>, where "the application of the cancelling principle is a characteristic of this text." Also, "it is not expected that the teacher should be confined to the forms set down in the book."[1.]

Jowle B. Thomas in his <u>Mental Arithmetic</u> says: "It is believed that much dislike for the study of arithmetic and much unnecessary discouragement have been occasioned by the abruptness from easy to difficult questions."[2.]

1. No. 7, Preface.
2. No. 35, Preface.

Fitter's Analysis of Written Arithmetic is the source of the statement that "pupils are usually given rules too fast. Mere illustration of the rule is not enough. There must be drill."[1]

John L. French says: "If the understanding is thoroughly reached, learning will take care of itself."[2]

These ideas are repeated again and again. They all demonstrate the general trend. Evidently, there was arising the belief that:

1. The content of the mathematical texts could be improved.
2. The mind of the child responds to certain treatment and fails to respond in other cases.
3. There must be a more careful and better graded study of the fundamentals.

That these ideas were being introduced to our teachers is highly important.

In the algebra texts, there was little improvement. These books are barren, dry and synthetic in treatment. However, one of these works suggested a mode of instruction which deserves our attention. T he idea that algebra might be presented orally seems to have occurred almost accidently in 1845 to David Tower. Tower was an instructor of the blind. Oral instruction to people thus afflicted was a necessity. It brought good results and Tower was encouraged to apply this method to the ordinary school. He used it in this manner: "After a question is read, call on

1. No. 15, Preface.
2. No. 16, Preface.

some one in the class to repeat it; on another, to state what is required; on a third for the data from which the answer is to be deduced"[1], so continuing through every step of the process of solution. To realize that this method is an improvement, the reader only has to glance back at the old copy book method. However, the following attitude was unfortunately the usual one: "To the student of mathematics, labor is discipline; and discipline, after all, is the true end of education."[2]

In the geometry text, we see improvements. We note a definite feeling that Euclid, in its original form, was not proper subject matter for beginning pupils. Mrs. Anna Cabot Lowell suggest that models of the different types of solids, should be presented to a beginning class and that these solids should be compared, described and contrasted. The class should then proceed to the study of lines and angles. Finally, the group should attempt Euclid.[3] On this score, Raumer in his <u>History of Pedagogy</u>, remarks, "Many educators are now agreed that Euclid's demonstrative course should be proceeded by one of intuitive nature." As reiterated in other texts, this pedagogist thought that reasoning in geometry should proceed from the study of geometrical form to propositions, not from definitions to demonstrated theorems.

1. No. 36, Preface.
2. No. 30, Preface.
3. No. 23

IV.

After Horace Mann, the Practice.

In the annual reports of the Massachusetts Board of Education, we get the most reliable information of what was being done in our schools and what the various secretaries and superintendents thought about the contemporary situation.

In 1850, we read, "Among the faults observable in the mode of teaching in the common schools is that of attaching more importance to words than to things. Another very general defect is that of treating the mind of a child too much like that of an adult. Another error of instruction is the neglect of the imagination... In mathematics the evil of the common course is that a child is put upon complicated processes before he can perform the ordinary operations on numbers with ease and accuracy. We often hear complaints against books in arithmetic, but what we need most is teachers; if they are right, there will be little trouble about books."[1]

As early as 1855, "an arbitrary inversion by which those subjects which are most important are most neglected. Arithmetic is studied year after year and recitations in it are made the test work of the school...while a selection of reading is not deemed worthy of study at all. Proficiency in arithmetic is thought to be better a criterion of the character of a school, and of the scholarships of the pupils than the other studies pursued."[2]

1. No. 1, 1850, page [illegible].
2. No. 2, 1855, pages 7[illegible] and 1[illegible]7.

In 1865, the same complaint, "No other study occupies so much time in the public schools as arithmetic. If, with so much of the school life given to arithmetic, our children do not become accomplished arithmeticians, a serious defect must exist somewhere."[1]

In 1872, "Arithmetic might be taught in a more practical manner than at present. One pile of wood, one stick of timber, or one lot of land, actually measured by the pupil and the results wrought out, or one transaction at the store, with the exchange made and the bill of goods written by the pupils, is worth a score of abstract examples which one never meets with outside the text book."[2]

In 1873, "Full success in this branch of mathematics will never be realized until there is complete emancipation from the slavery of textbooks. Our new curriculum which prescribes subjects rather than pages is already an improvement in this direction...In arithmetic, questions that simply test the endurance of the pupils by their length are no longer tests of knowledge. Nor are they of equal value as mental discipline. What we gain in the time of holding the attention is more than lost in intensity."[3]

In 1877, "The immediate goal to be reached in the study of mathematics is the acquisition of a useful amount of specific knowledge, with the ability to use it readily and express it correctly and clearly. We should bear in mind these principles:

1. No. 1, page 118, 1865.
2. No. 1, page [illegible]4, 187[illegible].
3. No. 1, page [illegible], 1[illegible]73.

1. In childhood, the emotional activities are greater than the mental activities.
2. Both single and related perceptions must be clear and distinct in order that the memory may do its proper work.
3. The imagination and reflective powers of children cannot live and thrive on abstractions.
4. Children, when their minds are acting freely and naturally, think and reason.
5. The mind is not educated until its power, not only of gaining knowledge without help, but also of applying and using its knowledge has been drawn out and made effective."[1]

From all these ideas, we conclude that our educators considered that the problems of organization, financing and housing in Massachusetts education were quite well solved and that the problems of classroom teaching occupied their interests. We see that there was at least a realization that the mind was of some consequence, that we should develop principles with which to get the best out of the mind of the individual and that these principles should be applied in the classroom. We also note the feeling that, although mathematics had grown into our curriculum, the time spent on it was not justified by any major achievements on the part of the average student. To remedy this situation, it was insisted that mathematics must deal with the objects and events of daily life. These arguments were to be continued in the period we are about to discuss--the scientific era.

1. No. 1, page 10, 1877

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1880-1910

THE BEGINNINGS OF THE SCIENTIFIC ATTITUDE.

1.

General Walker and the Boston Schoolmasters.

We have noticed in the previous section that there was almost universal dissatisfaction among schoolmasters as to the result of the teaching of mathematics in the contemporary curriculum. On this score, General Francis Walker, president of Massachusetts Institute of Technology, presented his suggestions to the Boston School Board and they were unanimously accepted. Here at last was something definite, in great contrast to the nebulous theorising which had proceeded. There were passed the following orders concerning the study of arithmetic:

1. Home lessons in arithmetic should be given out only exceptional cases.
2. The mensuration of the trapezoid and of the trapezium, of the pyramid, cone and sphere; compound interest, cube root, and its applications; equation or payments, exchange, similar surfaces, metric system, compound proportion and compound partnership, should not be included in the required course.
3. All exercises in fractions, commission and proportion should be confined to small numbers.
4. In practical problems, all exercises are to be avoided which any fairly intelligent and attentive child of the age concerned would find of considerable difficulty.

5. In oral arithmetic no racing should be permitted; but the dictation should be of moderate difficulty and rapidity.
6. The average time devoted to arithmetic throughout the primary and grammar school should be not more than three and a half hours per week.

Here General Walker gives his first consideration to the child, with only secondary emphasis on the teacher, the school or science. He felt that this was necessary to eliminate the poor results of mathematical instruction in this country. The cause of the difficulty was not unprepared teachers, abominable texts, or inferior teaching methods, but the fact that we had forgotten that school was made for the child.

After the publication of the above report, American educators began to notice the child-centered pedagogy of Europe. The result was that some of these pedagogists had great influence on future American education.

ii.

Herbart, Grube and Freubel.

Jean Frederic Herbart (1775-1841) and his precepts had long been accepted in Germany. His educational philosophy and his laws of instruction had been minutely followed. To Herbart there five unvarying steps of instruction: clearness, association, systematization, generalization and application. Like many philosophers he could not see that this order was much too unbending and only partially satisfactory. However, to the American teacher, Herbart was a God-send and his ideas were enthusiastically ushered into practice. In the mathematics class, this meant the application of the five steps of instruction and added emphasis on object teaching.

Herbart evolved an educational system for the higher grades. Frederich Froebel did this for the pre-school child. In his kindergarten, we have the first recognition of the "play" concept. Children were given tasks which would strengthen their bodies, excercise their senses and engage their awakening minds. Applied to the kindergarten, this idea was practicable, but it was later emphasized in all types of education.

Another European of note was Hans Grube. His method was heuristic, because it tried to make the pupil work instead of the teacher. He advises the teacher to have the first grade child spend his time with numbers one to ten only. The process is slow, but when the year is over, the pupil understands the numbers one to ten, the concepts involved and all possible combinations of them. He is well grounded for the work of the second year which is concentrated on the numbers ten to one hundred.

The principles of Herbart, Froebel and Grube made their American debut in new texts about 1890. They were the basis of the splendid efforts of Dewey and his followers. For instance, Ellen Barton, in her Language Lessons in Arithmetic, devised a method "similar to Grube but not carried to his extreme of useless repetition and mechanism."[1] George Atwood initiated his "spiral arithmetic" which can summarized in one word "review". As he says, "The spiral plan consists in daily reviewing what had been taught before and then introducing new material."[2] All of our progressive texts, at least, insisted that the teacher appeal to the child through the senses, and that the order be philosophical, but not necessarily logical. This was helpful in the early grades, but not helpful in stirring our secondary school teachers from their long neglect of algebra and geometry. By neglect, the writer does not mean that these teachers had not assiduously taught algebra and geometry for years, even overemphasized its importance, but there had been no improvements that could favorably compare with the work that had been done in arithmetic. To satisfy Herbartian demands there arose concrete or intuitional geometry. It was known by other names—observational, constructive or demonstrative geometry, which are merely variations of the same thing. Later, it was to be elaborated on, but now it was suggested as

1. No. 4, Introduction.
2. No. 1, Introduction.

an introduction to the course in formal geometry.[1] In these new attempts, the beginning geometrician was allowed to measure with compass and rule. He built and worked with geometric models. He was definitely presented "objects before ideas".

In algebra, J. . McDonald remarked that "the critical period is the first two months. Here the teaching was usually defective and vague expressions such as "a over b" or "transposed" were thrust upon an unwilling pupil."[2] As secretary of the Board of Education, Mr. Dickinson often stressed this desirability of beginning algebra with concrete cases, no juggling of abstract symbols, but an attempt to impress the student with the worth and reality of algebra.

.No. 13
2. Annual Report Board of Education, 1891, page 317.

II.

John Dewey.

At the turn of the century, the leader of those men who were finally to reorganize American education came into prominence. He was John Dewey, born in Burlington, Vermont, graduated from the University of Vermont and John Hopkins University in 1884. He taught philosophy at the Universities of Minnesota, Michigan and Chicago. At the latter place, he first won national fame as director of the School of Education. Dewey is par excellence the educator of democracy and the industrial era. He was highly cognizant of the social and the economic factors which made up this twentieth century, as well as the educational ones. Here was a new country bursting with wealth, independence and imperialism. There were many jobs to be done and youth needed to be trained to fill these jobs efficiently. Dewey saw no better place for this than in the schools. Why, he asked, should we give students but a classical education and then leave them to wallow disconsolately in the mire of confusion after they quit school? Dewey had no patience with the classical approach and general concept of the desirability of a "good" classical foundation. His dictum was: "Education is life", which to him meant that the schools must better acquaint youth with the world in which we are living. Education must be practical. But how was this to be effected? Were we to take the practical facts and teach them heuristically? Before answering this, Dewey chose to investigate the character of the learning process. To him learning was merely the ability to recall specific experiences, while the experience which was most vital and therefore the most easily remembered was "doing." The only obvious conclusion was that the student should learn by "doing", not learning facts but understanding principles and processes.

John Perry of England was another who evolved ideas similar to Dewey's which were soon to spread throughout our schools. He remarked, "I have said that it is usefulness which must determine what subjects ought to be taught to children and in what ways. I say that the old Alexandrian method is bad. Perhaps the mathematicians will forgive my impertinence in saying that even for the boy who is likely to become a great mathematician that method is bad. Even so, we have ten thousand Toms, Dicks, and Harrys mentally destroyed for the sake of a great mathematician. Rather let Tom make his own discoveries, let him educate himself through his former experience. It is a social wrong under the name of pure science to force such minds into paths having next to no meaning and consequently leading next to nowhere."[1.]

Somewhat later, Simon Newcomb suggested: "There is very educational value in very short and very easy lessons. This is one of the main features I am trying to develop."[2.] In this well-meant remark of Mr. Newcomb, we get some inkling of how the Dewey concept might be misunderstood and overemphasized to a harmful degree by the glorification of method which would be the exact opposite of that of the disciplinarian.

1. No. 21, page 155.
2. No. 19, page 86.

IV.

The Application of Dewey's Ideas to Mathematics

However, in 1908, Mr. Dewey found his ideas definitely accepted by a commission appointed by the International Congress at Rome. They recommended:

1. The elimination of non-essential topics.
2. That in the seventh, eighth and ninth school years, pupils should acquire such knowledge of mathematics as the average well-educated person needs to possess for purposes of general information.
3. That this range of general information should include the generally usable part of intuitive geometry, beginning algebra, simple formulas, the meaning of trigonometry and the application of these in commerce, industry and the arts.
4. That this mathematical material should be required through the ninth year.
5. That, after the ninth grade, mathematics should be elective.[1]

At this time, few realized what a really strategic move this was for the commission. Most of our so-called mathematicians merely showed resistance to these new trends, not appreciating that mathematics was a perfect target for the pragmatism of Mr. Dewey. As the subject was commonly taught, it certainly was not practical

1. No. 7, page 293.

and it certainly was not life. However, the natural reaction often occurred, that of rejecting Mr. Dewey as a radical and raving about the disciplinary and moral values of mathematics.

But what of the doctrine of transfer? For years, it had been the mathematics teacher's boast that a lad who had mastered their science would be a superior student in other fields, because all his mental powers would have so thoroughly developed. Messrs. Thorndike and Woodworth were teaching otherwise. In their studies at Columbia in 1901, where highly related mental abilities were studied, such as training in estimating areas, length, sizes, observation and perception of various forms, they were forced to conclude: "It seems likely that spreads of practice occur only where identical elements are concerned in the influencing and influenced functions. There are no general functions such as attention, powers of observation, powers of observation and accuracy."[1]

This was undoubtedly the greatest solar-plexus blow that mathematics had received in the history of our country. Educators looked back upon centries of misdirected effort. The effects, after twenty-five years, have been, in the writer's opinion entirely purgative. Faced with the choice of scientific disapproval or fitting school mathematics to the new era, educators made the proper and wise decision. They were to attempt a reorganization of their field. Many noteworthy suggestions came forward as to the means by which this might be effected.

In arithmetic, there were vast improvements in algorism "Additive subtraction" was a real contribution, and the Austrian method of division was a boon to both teacher and pupil. In

1. Nr. 7, plate 2. in back of cover.

multiplication, instead of presenting the comparatively hard table of sevens in its logical order, many teachers presented it last because children found it harder to grasp. Thus was the new arithmetic more efficiently and easily taught. Not only that, but any subject which could be treated quantitatively and was familiar in the life of the average child, came to be considered good material for the mathematics lesson.

In the grammar school or what was in some places the junior high school, the advances were even more startling. Pupils were learning the fundamentals of banking, finance and national affairs in the mathematics class.

For the high school, Miss Edith Long proposed a system which was tried out in some Massachusetts cities, and which is still in the process of experimentation. This was a plan for correlated algebra, geometry and physics. "The introduction of mathematics under these conditions is but the clothing of fairly well grounded information in algebraic and geometric language"[1], Miss Long explains. The amalgamated course was introduced daily for two hours, with the principles furnishing the basic substance of the instruction. These principles were arrived at inductively through laboratory experiments, then problems were solved as application of the physical principle, which finally resulted in the formulation of mathematical laws.

Professor Newcomb set forth a "system of visible and graphical representation by which the advantage of apprehending mathematical ideas in a concrete form should be gained. After learning

1. No. 13, page 48.

fundamental processes, we extend operations to continuous quantity by lines and areas on paper or on the black board."[1] What could be more simple? But until that time, few teachers were interested in graphs. Newcomb, in his search for concreteness in method stumbled on this as a good means toward this goal. Graphs were being presented to our citizenry in newspapers and magazines. They were fundamentally mathematical in nature. Therefore it seemed reasonable to insert them in algebra courses. This was done and a topic was added to the curriculum which has since been the delight of most secondary school students.

As to algebra, educators had been shown that performance was poor and that specific transfer was impossible. It seemed advisable to strive for the inculcation of general concepts. William Betz remarked, "I do not believe that the power to solve problems is the greatest aim of a course of elementary algebra. We are seeking to form a habit of thinking in general numbers."[2]

Certainly during this period, the teaching of mathematics had improved. Dewey had been heeded. The very foundation of our mathematics curriculum had changed. Steps had been taken toward concreteness and practicality. Psychological research had been encouraged. Was this to continue?

1. No. 20, page 32?.
2. N. E. A. Journal, 1908, page 6?9.

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1910-1930

THE SCIENTIFIC ERA.

I.

Psychological Testing.

About 1910, people, in general, and psychologists, in particular, began to realize that vast changes had taken place in our methodology and in our curriculum. And far be it for Americans to do a thing halfway. A statistical method was evolved and was being utilized in certain branches of psychological research. Why not use it to measure the results of the new pedagogical theories? This question must have occurred to them, and seemed almost divine in origin because it was certainly seized upon with alacrity. Here, at least, was a chance to be scientific. No longer would they depend upon personal opinion or the traditional examination. No, there would be accurate measurements. Klapper said, "Unless we use scientific standards, we have no means of coming to a conclusion worthy of respect on some very vital problems in school work."[1]

This precept was generally accepted and every new test was greeted enthusiastically. We shall review the content and purpose of a few of these. Probably, the most widely used were the Courtis Arithmetic Tests, consisting of graded examples in the fundamental operations. Each test was to be administered for a specific time period. Mr. Courtis attempts to determine from the scores made by many children a goal or a standard of accomplishment for each grade which teachers might accept the country over,

1. No. 7, page 338.

thus furnishing themselves with a valuable basis for comparison.

Another was the Monroe Diagnostic Tests in arithmetic. Here the purpose is quite simple. Learning is analysed as a series of habits, so that the faulty learning process he has failed to grasp. The teacher is often presented with the problem of just what phase of addition, for example is bothering Johnny. In his test, Professor Monroe begins with very simple exercises, follows these with exercises of more complexity and finally incorporates all the adding difficulties in one problem. Hence the teacher is enabled to discover just where Johnny is having his trouble.

Similar to the Munroe tests are the so-called progress tests, which are designed to measure standing, progress and ability of the individual student in relation to the rest of the class.

Of the reasoning tests, Professor Stone's is the best known. It should be noted that he does not seek to measure reasoning power as such. Rather he wants to measure individual ability to see the steps of some common word problems. After analysing the processes necessary to the solution of a specific problem, he presents examples which demonstrate each of them, thereby furnishing a guide to the crux of the individual's difficulty.

In algebra, there are the Hugg-Clark tests, which present graded work designed to test the students with regard to several formal algebraic operations.

The Holtz Algebra Scales make use of problems whose difficulties have been determined not through analysis but through a statistical study of tests results. They have been administered to many thousands so that the teacher may well compare her class with the norm

In geometry, the teacher has an interesting problem with which to wrest. Particularly in 1910-1925, there were many different kinds of geometry: practical shop work, intuitive geometry, mechanical drawing, formal proving of theorems and working of problems and correlated work with other sciences. Which of these was valuable? Were they all of equal value? Dr. Minnick hoped that these questions could be answered by giving people with these various types of training the Minnick Geometry Test which was to show how much true geometric ability the student had achieved. Dr. Minnick assumed that this would be done, if he measured:

1. The ability to draw a figure for a theorem.
2. The ability to state the hypothesis and conclusion accurately in terms of the figure.
3. The ability to recall additional known facts concerning the figure.
4. The ability to select from all available facts those necessary for proof and to arrange them, so as to arrive at the desired conclusion.

The reasoning tests were conservative attempts at measurements. Those tests which tried to measure the "native" mathematical ability of the individual must be called radical. No one denies, neverthless, that if we could measure this ability it would be a magnificent achievement. Roger's Test of Mathematical Ability is significant in this field. He assumes that he is accomplishing his purpose by testing Algebraic Computation, Superposition, Mixed Relation and Language. In the sections on algebra and geometry, certain truths are first stated which the student will need if he is to successfully complete the examination. Here, we have at least a faint attempt to lessen environmental influences and give the students a chance to start at par.

Far from being merely the research of specialists, these tests were applied widely throughout Massachusetts. That they were appreciated we may be sure when we read: "When clairvoyance and divination were for the first time in history replaced by science, by the inductive experimental method, how few were conscious of the momentousness of the event that had taken place. How few realized that the turning point in the history of mankind had been reached."[1]

II.

Transfer and Interest.

Along with measuring abilities and results there was still the problem of transfer. After the study of mathematics, was there transfer of training or was there not? The answering of this question was a very complicated task, where the importance of partial studies was apt to be overemphasized. On looking over the experimental data on this field, we are at no loss to understand why it was that an argument arose in the first place. Thorndike reported no transfer. Judd reported some. Nellie

1. No. 12, page 416.

Hewins reported that there was a decided transfer from the mathematical to the biological sciences. Bagley reported that there was no transfer from neatness in mathematics to neatness in other subjects. Hewins found no transfer from reasoning ability in mathematics to reasoning ability in law and business. Rietz and Shaw found a decided transfer of ability from mathematics to ability in other subjects. And so it went, each new report contradicting the last. When we consider Professor Coutis's comment on such remarks in general, we are inclined to take them all with a grain of salt. He says that no such studies are valid unless

1. the criteria by which the nature of the abilities measured by the test were set, are satisfactory,
2. the reliability of the tests used have been scientifically demonstrated,
3. the development of general ability in the specific field has been proved to exist before the test for transfer was made.[1]

However, the general educational population, by and large, took violent sides in the argument. There were a few who were stern disciplinarians and said that "the fact that a mathematical piece of work must be right or wrong and, if wrong, the mistake must be discovered and corrected by the student; and that a false step in a demonstration must lead to a false result, and that correct reasoning must lead to correct result is so clear an analogy to moral life that the student gets an excellent training in habits of honesty and moral uprightness."[2] More were willing to suggest that mathematics should not stay in the curriculum unless

1. No. 3, page 507.

it could justify itself by obeying the doctrine of interests. According to them, Mathematics must be made interesting at all costs.

Clark and Otis suggest that there should be an end to the recitation--discussions are to be the order of the day and they emphasize that: "We have come to see, in the light of new knowledge of the facts and principles of mental and moral growth, presented by genetic psychology, that what a student enjoys he profits by."[1] Lennes and Jenkins hope that each teacher will realize the necessity for:

1. Selection and organization of matter to bring simplicity.
2. Derivation and application to achieve practicality.
3. Motivation through interest.[2]

Through games, junior high school is to become as interesting as marbles. The children would profit by it because interest is a measure of profit. Taylor suggests that in high school, it might be well to eliminate the text book until the advanced work.

All these concepts were not merely theories but went into actual practice in the school systems of some localities. The whole movement culminated in the Dalton Plan and the Progressive Schools.

1. No. 2, Preface.
2. No. [illegible], Preface.

iii.

The Dalton Plan.

About 1920, Miss Helen Parkhurst and Mr. Ernest Jackman started a daring experiment at the high school at Dalton, Massachusetts. Their first consideration was the interest and advancement of the child. His work was adjusted by himself as to speed and daily quantity. Class progress was measured by the median rate of the individuals in the class. The school was graded, but without "grade rooms" or "home rooms"--Jackman preferred to call them subject laboratories. Each teacher specialized in one subject and managed a subject laboratory for all student above the fourth grade.

There was no time schedule, no bell to summon the child from room to another. The student apportioned his time independently, deciding in which subject laboratory he would work at any particular period. He entered and left the laboratories as he pleased. He was encouraged to talk with the other students and to work with them.

His assignments were in the form of monthly contracts, large bunches of work in a subject, where the individual had displayed special interest. There were no marks but the pupil's progress was recorded on graphs at the front of each laboratory.

Here mathematics was taught as an informational subject, and in such a manner as to give interest and variety to the learner. The use of apparatus was emphasized, because nothing was taught unless it could be formulated into a principle which could be demonstrated and applied to daily life. But were interest, variety and independence the only ingredients and characteristics of an acceptable pedagogy? This was the next question which educators had to answer.

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SUMMARY AND CONCLUSION REGARDING THE PRESENT AND THE FUTURE OF ELEMENTARY INSTRUCTION IN MATHEMATICS.

I.

Controversial Questions.

During the last ten years that there have been changes, few will deny. The versatility and the variability of these changes are particularly noticeable, which in turn have brought many important educational questions to the fore, clamoring for answers. Among them are these:

1. Is mathematics necessary (particularly as a part of formal instruction)? There are many among us, evidently abolitionists or destructionists by nature, who maintain that mathematics should be forcibly ejected from our curriculum; that it hampers the creative ability of the individual; that it is needed only be the relatively few who become engineers or scientists.
2. Should mathematics be taught as an informational subject? There are others who are vastly aware of the complexities of modern life, and plead that if we fail to use exclusively modern factual material there is no excuse for mathematics at all. They realize that every day the average citizen is called upon to know more facts, if he is to comport himself in a manner pleasing to his friends and profitable to himself.
3. Should the child be permitted to use his own creative ability to work out his own propositions and, in general,

his own mathematical concepts? It is an admitted fact that we remember well academic material which we "discover" for ourselves and some go so far as to say that it is only that type of material which is retained; that there is no sense in forcing the individual into specific paths of learning, because he fails to respond to such treatment.

4. Is there any particular disciplinary value in mathematics? As we have seen, this is a topic which brings forth much animated argument. Naturally, discipline in mathematics is of little value if it is wholly untransferable to other activities. Even if it is, some say that a disciplinary aim is not only unimportant but exceedingly deleterious. Those who hold this belief insist that no student thrives under discipline and that he should live in an environment full of encouragement.

5. Shall we depend upon psychological testing as the only valid means of measuring ability, progress and interest? The psychologists would tell us to abandon all atempts toward philosophical measurement. Man must be treated like a machine and given tests based upon psychological research if we hope for valuable results. But the opinions of what is desirable differ so much among the psychologists that the problem becomes even more difficult.

6. Should we train for life in terms of an occupation or in terms of life as a whole? In this age of increased unemployment, when the school graduate, no matter what his worth, is beset with numerous defficulties, some think that our

first consideration should be that of preparing the young American to take his place in the industrial world. If he is well situated occupationally he is then in a position to adapt himself to life generally. With a similar premise, some would emphasize leisure time education, training for life in its broadest sense and letting the student later adjust himself to vocational problems.

There are only a few of the more obvious questions and it has been the finding of the writer that we are in a position to answer them intelligently only when we fulfill two specific conditions: (1) We must review the development of mathematics in this country during the last two centuries. (2) We must develop a philosophy of education. The first, the writer has tried to do in the preceding pages. It is the second point which we shall now consider.

ii.

A Philosophy of Education

That we need a philosophy of education is apparent even to the layman. The multitude of contradictory views on education is enough to convince us that many educators have no vision and idea where education is going, should go, or why it should travel in any particular direction. How are we to determine the matter? Most of us moderns would say scientifically, demonstrating the present day obsession for anything that can be labelled science. What is science? Is it something which is inherently valid in itself, and thereby forordained as a proper measuring stick and decider of values? Many would have us think so, but actually science is undershot with as many assumptions as philosophy. This is overwhelmingly true in the so-called sciences of sociology and psychology. Take, for example, one aspect of psychology as related to education--mental testing. When we consider the matter seriously, we note these assumptions running through the whole program:

1. That we can measure the ability, aptitude or knowledge by behavior.
2. That the capacity for different activities is ascertainable.
3. That the factors of chance do not operate.
4. That the emotional status of the individual will not influence performance.

Mental testing, then, is not scientific and never will be. It is in the same class as the statistics which measure how many and assume that they have measured how desirable. We can never tell what is a good education for the carpenter, farmer or professor by job analysis. As Professor Counts remarked: "The goals of education cannot be determined by scientific method."

Attempting, then, to ascertain the goals of education not scientifically, but philosophically, we seek to answer the question, "What purpose should education fill?" But ultimately education should serve the individual, so we must ask, "What should the individual seek in life?" To me it seems that he should seek his own welfare and happiness through cooperation with the desires of the social group. The best way to do this would be to properly participate in all our social institutions. The inevitable conclusion follows, that the goal of education should be the enabling of the individual to lead such a life--<u>to prepare for the well-balanced participation in the social institutions</u>.

But what is the nature of our institutions? Are they entities which are constant in character? Are they as they were yesterday? Will they be the same tomorrow as they are today? Only a partial reflection will convince us that the institutions, the family, local community, state, industry, church, school, press, standard of living and customary recreations--all are the most variable things possible. We are faced with not the possibility, but the probability of a very changed set of institutions within a few years. We have no way of prophesying their characteristics, and we are to attempt to educate youth for the proper activities. What are we to do?

In the writer's opinion, we cannot with equanimity overlook a possible telic function of education. There must be an agreement as to what we are working for, what kind of a society we desire and drive toward that end. Of course, for this idea to come into fruition involves not the work of years, but of centuries. It involves a national unity which we now lack, a conviction in the desirability of social planning, a working system of civil service examinations whereby every government office-holder would be subjected to the most careful scrutiny. Our educators and our statesman must have social vision and proper understanding of the learning process to strive for this.

So few realize that in the educative process itself we are not considering the process of formal education. The great mass of the education of the individual is not gained in school, but in the several other institutions. Therefore the curriculum of the school should be made up of those subjects, that informational material, that "disciplinary" material which the student would not otherwise have the opportunity to acquire in the other institutions. This will eliminate much of the fal-de-rol that is now in our curriculum--religious training, too much physical education, too much "patriotic" emphasis of the past glory of our country. It would add other items--an exhaustive study of internal relationships, the essentials of psychology, better appreciation of the arts.

Above all, education must not enhance individual difference, but minimize them, strive for understanding of the common lot of man, and the common purpose which we serve. We must minimize the advantages of the white collar class, and through a thoroughly cultural education, bring an appreciation of the more humble occupation of the farmer, and ordinary worker. After all, all of us cannot be executives, minor or otherwise, and the desire of the group to attain such positions brings only unhappiness and impoverishment to the mass of society.

ii.

The Place of Arithmetic.

Is arithmetic to be included in our future curriculum? It certainly is, not only by tradition but by choice. In the daily work and play of any person, there is and will be a constant need for the application of arithmetic. But what is to be the aim in our teaching of this subject? To the writer's mind there four very fundamental principles:

1. To give the pupils a knowledge of the fundamental processes and terms necessary to meet the needs of life.
2. To develop in pupils the power to reason accurately in meeting the problems that are likely to occur in business transactions and the practical affairs of life.
3. To develop skill, accuracy and rapidity in dealing with numbers suitable to the age and experience of the students.

4. To aid pupils in interpreting and comprehending the quantitative problems that arise in the world's activities.

To realize these aims, the writer feels it is advisable to proceed in a manner similar to the following one:

First, the teacher must decide whether the pupils are to be subdivided into graded divisions. In a large school, this will probably be advisable and will save a lot of time and energy on the part of the instructor. However, in the average or small school, this is certainly not the case. By grading, the individual differences are aggravated, where they should be lessened. The fundamental principle that nine-tenths of learning is a process of passive mentation is ignored. With this in mind, however, we see that in an ungraded group, there is the greatest possible opportunity for the dull to learn from the bright and, with the skilled teacher, for the encouragement of the superior pupils.

The study of arithmetic should begin early. The writer is convinced that long passive inurement to a particular environment is much more effective than a short period of intensive study in that same environment. When begun at this early age, or at any age, there is an absolute necessity in making the work "interesting". To gain this interest, the teacher bases every part of her instruction on the past experiences of the pupil. This means that, for the lower grades, particularly, much concrete material and varied objective work. By varied, is meant a frequent change of the object material used and decidedly more attention to visual or picture aids. In other words, the work must be motivated. The problems should be those which the student has met or will meet in the near future. We have seen the failure in the past was at least partially due to ignorance of these principles and the presentation of material which had no initial meaning to the student.

Another means of motivation is through construction work, projects and games. But here let me issue a word of warning. Nothing is more stupid than an overdose of this motivation or "play" theory. Education is not and should not be play. Furthermore, there is no psychological or logical basis which would make this seem desirable. We do not necessarily learn or remember what we enjoy or forget what we dislike. A glance at our own personal experiences will justify that remark. Then we must also ask ourselves, "In the future participation of the individual in the social institution, will his experiences be a series of games or motivated performances?" The answer is a very firm "no". In order to train for the disappointments, hardships and difficulties of life, we must insist on attention and hard work, although some of it may be slightly uninteresting to the student.

In arithmetic, this means that there will be a sensible amount of drill or purposive repetition. Some maintain that increased repetition does not bring increased mental imagery, but that learning comes through the Gestalt concept of "insight". However, there seems to be more evidence which favors drill work. It must be short, snappy, direct and not too easy to be effective.

Although there are individual differences in any group, the good arithmetic teacher will keep each student occupied at his maximum capacity, and heed the frequent need of remedial measures. Instruction for individual difficulties must be given.

For the child who is having great difficulty, for unaccountable reasons, a home-made, analytical, diagnostic test is as good as anything.

Probably the greatest error committed by many teachers is that of offering disconnected (as far as the students are concerned) bits of information. What he should be teaching is daily life clothed in quantitative language. He must move slowly and cautiously, basing each step on information which is so well known to the class as to have become second-nature. For example, teaching the multiplication tables alone and as such is a waste of time and a method based on fallacious assumptions. So it is ridiculous to employ positional learning if the multiplication table. The student should remember that 8 x 7 = 56, not by its relationship with 8 x 8 or 8 x 6, but because it is a fact which he has used in his applied work.

In large groups, the standard tests are of use, but should be used sparingly. They will measure, principally, the average work of the class. In a small group, they are a waste of time since any interpretation based upon an individual score would be quite worthless. It is only when these tests are coupled with several other kinds of judgment (teacher's and other's) that the individual results are at all significant.

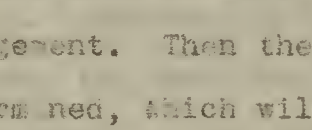
IV.

The Place of Geometry and Algebra.

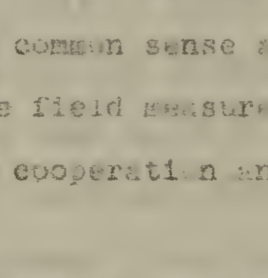
As in arithmetic long inurement to geometry and algebra, an early instruction in the subjects is advisable. The sees no reason why there should not be continued encouragement for its induction into the junior high school.

Successfully introducing a course in plane geometry is about the hardest thing known to man in my humble opinion. Here we are introducing to a group of adolescents an elaborate science whose very name they do not understand. It is difficult in the first place, because it is practically impossible to convince a group of fourteen year olds, that they should study anything whatever. This difficulty must be overcome by making the work of the first few weeks in itself be explanation enough to the fact that studying geometry is absolutely essential to their future happiness. To do this, give them compasses and rulers, use no text, and make the work as concrete as possible. Any normal child is capable of "seeing" the first few theorems, the axioms and postulates with no more than his own common sense and the teacher's encouragement. Then there are simple field measures, areas to be determined, which will arouse the cooperation and interest of any class.

Another difficulty is that it seems to the student manifestly absurd to prove what you can plainly see with your own two eyes. Why should he prove the angles equal when he can see that they are equal? This can be taken care of by demonstrating to the student the faultiness of his own sense of judgment and vision by having him compare such diagrams as these:



Which is the longer line 12 or 1'2'?



Which is a line straight ab, ab' or ab''?

A third difficulty is the matter definitions which must be introduced to the student to provide a working vocabulary, Memorization, in the first stages of geometric learning, is sheer barbarism, but a series of exercises using these terms both singly and mixed is quite effective.

If the teacher surmounts these early difficulties, she will find her geometry class less of a problem every day. After the first crucial weeks are over, there comes the beginnings of informal geometry. Here the method should be overwhelmingly inductive. Given a clear diagram, understanding of the language--any normal child can prove for himself most of the propositions in plane geometry. Naturally, it would not do for every student in the class to have his own private proof, so after the class has considered the various methods, they should choose the best one, adopt it for class use and record it in their mathematics notebook. Besides, it is often wise for the students to show themselves the need for entering upon the proof of a new theorem. For instance, if the teacher desires the class to learn the meaning and proof of "Two triangles are congruent if two angles and the included side of one are respectively equal to two angles and the included side of the other", she should present a problem like this:

The students will see that if the proposition were true, they could easily solve the problem. At this point, the students are willing to try their hand at proving the above quoted proposition, and do not feel that they are being driven to prove something for which there is no actual need. A continuation of this general method throughout the course, where every fact is needed before it is learned, and applied many times afterwards, should make the whole program a success.

In algebra, the situation is quite different in that the transition from arithmetic is distinctly easier. Algebra, is, after all, merely generalized arithmetic and the class can be easily convinced of the superiority of this shortcut method. What teachers are inclined to forget is the fundamental purpose of algebra in the curriculum. First, it should teach methods of solving equations and secondly, it should train the student to think in general numbers. It is certainly not its purpose to train agile jugglers of mathematical symbolism. If we keep this steadfastly in mind, and constantly in practice, we cannot go far wrong. Solving equations should involve, of course, much emphasis on the function concept. This can be given concrete embodiment in terms of everyday experience and visual interpretation through graphs. This part of algebra is exceedingly practical since the understanding of graphical language is a prerequisite to intelligent reading. However, except to those who are to study mathematics further, algebra cannot be said to have much of practical value. What value there lies in the opportunity which this subject affords for close hard thinking is admitted.

This is something to which the student must habituate himself because life will many times require such efforts of him.

The usual algebra problems are far from satisfactory. Why cannot algebraic principles be demonstrated in problems of an interesting nature rather than in those which deal exclusively with A and B, trains, boat races and emptying pipes? The material could be gathered from modern agriculture, physics, chemistry, geography, banking, finance and politics. The student can appreciate subject matter such as this, because it is brought to his attention every day of his life.

The writer has formulated what in her opinion are the principles by which mathematics should be taught to the present day boy and girl. Many of them she has already tried to her own satisfaction in the class room. If upon trial the rest of these principles prove unsatisfactory, she will have an adequate basis for rationally changing her point of view. She should be a better mathematics teacher and thus fulfill the main purpose of this thesis.

THE END

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